Multi-saddle-point approximation for pulse propagation in resonant tunneling

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Poles of the scattering matrix in the complex energy plane play an important role for pulse propagation through a resonant tunneling system. Therefore, we employ a special variant of the method of steepest descent to calculate the transmitted pulse. In this variant, the parameters defining the transmission resonances and the shape of the incident pulse are taken into account for the evaluation of the saddle points. As a result, we obtain a multi-saddle-point approximation (MSA) with a number of saddle points which are specific for a given combination of pulse and resonance. This is in contrast to the usual variant of the saddle point approximation [single-saddle-point approximation (SSA)] in which only the phase of the free time evolution operator is made stationary, leading to a single saddle point in the resonant tunneling problem. We apply our multi-saddle-point method to an incident Gaussian pulse comparing with the results of the SSA and the analytical solution. Apart from quantitative advantages of the MSA, in general, it is found that in contrast to the SSA, the MSA is capable of describing the center of the transmitted pulse for a narrow resonance.

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I. INTRODUCTION

The transmission of wave packets through a resonant quantum system (QS) is a long standing problem. It is interesting from either the theoretical or practical point of view. In particular, its solution could find applications in the description of resonant pulse transmission through various devices such as double barrier diodes, quantum dots, and resonant tunneling transistors.[1,2] For effectively one-dimensional systems, pulse propagation in resonant tunneling has been investigated in detail numerically[1,3–6] and in a number of cases analytically[7–10] (for reviews of tunneling transport in general, see Refs. 1 and 11). It is commonly believed that the problem has no general solution because the transmitted pulse depends strongly on the complicated relations between the parameters defining the incident pulse (the width, the position of the center of the pulse in the momentum space, the pulse form) and the parameters defining the QS (energy levels, distances between them, widths of levels). To find the generic properties of pulse transmission, nevertheless, it is interesting to study analytical solutions found in special cases and to compare them to solutions resulting from more general approximation schemes.

In the present paper, we develop further a method describing pulse transmission through a resonant QS described in Refs. 12–14. Here, a resonant nanosemiconductor device is represented by a single Breit-Wigner resonance or a single Fano resonance in the transmission $S(k)$. The incident pulse is represented by its associated wave function in k space. We apply the method of steepest descent to find the transmitted pulse for an arbitrary incident pulse. As a first application, we consider an incident Gaussian pulse interacting with one or two resonance poles of the S matrix. A Gaussian pulse is taken, first, because in this case, analytical results exist,[13] which we use to verify our saddle point method. Second, it is particularly easy to relate the Gaussian pulse to an experimental incident current pulse.

The method of steepest descent has been used in a number of transient problems closely related to the one we consider. Usually, the relevant saddle points are determined by the stationary conditions applied to the free phase only [single-saddle-point approximation (SSA), see Eq. (13)] while possible contributions of poles have to be taken into account separately.[15–18] This procedure depends either on the type of the initial state or the structure of QS. In the version of the steepest descent method, we propose the calculation of the saddle points including information about the initial wave packet and the tunneling structure as well. This leads to a number of additional saddle points [multi-saddle-point approximation (MSA), see Eq. (11)] representing the contributions of poles in the transmission amplitude or even in the Fourier transform of the incident pulse.[19,20] Being joined with a pass to suitable dimensionless variables, this gives a general solution for the pulse transmission problem. We find for our particular application that in contrast to the SSA, which is a good approximation for wide energy levels, the MSA is well applicable for narrow resonance states also and, in addition, it can describe two important phenomena found in the analytic result for the transmitted pulse: First, the decaying-state type of transmitted pulse that results for a narrow resonance and, second, characteristic oscillations superimposed to the decaying-state signal.

The paper is organized as follows. In Sec. II, the multi-saddle-point method is developed. In Sec. III, the transmission of the Gaussian packet is investigated and analytic and numeric results are presented. The exact and the asymptotic expressions are compared for different types of resonance levels. In Sec. IV, the conclusion and the comparison of the SSA and the MSA method are given.

II. MULTI-SADDLE-POINT METHOD

We consider a one-dimensional wave packet, characterized by the wave function $\psi(x,t)$, which is incident for negative times moving in the positive direction along the x axis [see Fig. 1]. The group velocity of the incident pulse is $v_0=\frac{\hbar k_0}{m}$, where m is the effective mass of the particle, k_0 is its

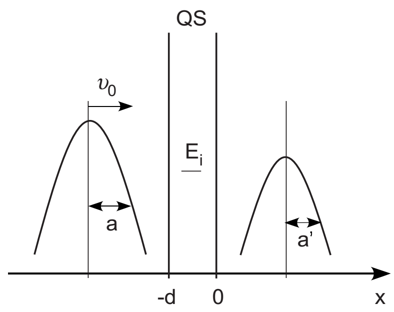

FIG. 1. Schematic representation of the resonant transmission of an incident pulse.

center of mass in the momentum space, and a is its width in real space. The space coordinate is chosen so that the scattering potential is confined to the interval $x \in [-d,0]$, vanishing outside the interval. The time coordinate is taken so that at $t=0$, the center of the unperturbed incident pulse is located at $x=0$. We are interested in the transmitted amplitude in the domain $x>0$ for the time moments $t>0$. It can be described by the following standard expression:

$$\psi(x>0,t) = \int_{-\infty}^{\infty} \frac{dk}{\sqrt{2\pi}} \psi_0(k) S(k) \exp\left[ikx - i\frac{\hbar}{2m}k^2 t\right], \quad (1)$$

where $\psi_0(k)$ is the wave function of the incident pulse in k space. Here, we assume that the confinement of $\psi_0(k)$ around k_0 is strong so that essential contributions to the integral arise from positive k only.

Thus, the scattering problem is reduced to the calculation of this integral. Of course, this cannot be done analytically in general and some approximation schemes must be used. The most adequate is an asymptotic expansion. Usually, a stationary phase method with the phase of free motion is applied. However, this method requires the factor at the oscillating exponent to be smooth.[21] This property fails in the vicinity of the pole position for a narrow resonance. So, additional considerations have to be employed.[22–24] To overcome this difficulty, we propose to proceed with the special form of the steepest descent method which includes in a natural way information about an arbitrary resonance (its position in the complex momentum plain and the width). In this representation, information about the resonance is incorporated in the phase factor which is a complex valued function having a shape maximum for a narrow resonance. Then the integral can be approximated well by the asymptotic expression given by the standard saddle point formula. This is so because the narrower the resonance is, the better all the necessary conditions for the applicability of our method are fulfilled in the vicinity of the saddle point in the momentum space. In fact, in this way, we invert the obvious disadvantage of the stationary phase method into the advantage given by the saddle point consideration. That actually is possible due to the transition into the complex k plain.

Furthermore, we use an expression for the transmission amplitude as given by

$$S(k) = \sum_{i=1}^{N} \left[S_0^i \frac{i\frac{\Gamma_i}{2}}{k - k_i + i\frac{\Gamma_i}{2}} \right] + S_b, \quad (2)$$

which is discussed in Appendix A. In the representation in Eq. (2), it is assumed that in the relevant k range where $\psi_0(k)$ is non-negligible, $S(k)$ is composed of a background term S_b and of the contributions of N resonances $i=1,\ldots,N$ of weight S_0^i. These resonances are located in the complex k plane at $K_i=k_i-i\Gamma_i/2$.

In a scattering problem, one is usually interested in the transmitted amplitude at long time intervals. To investigate this case, we use the saddle point method which allows us to calculate analytically the asymptotic value of integrals of the form $\int_C f(z)e^{\lambda g(z)}dz$ for $\lambda \to \infty$, where C is a given contour in the complex plane. If the integral converges and the contour of integration can be deformed to the steepest descent path, the leading term of the integral is given by the expression[21]

$$\int_C f(z)e^{\lambda g(z)}dz \sim \sum_k \left[f(z_k)e^{\lambda g(z_k)} \sqrt{-\frac{2\pi}{\lambda g^{(2)}(z_k)}} \right], \quad \lambda \to \infty, \quad (3)$$

where z_k is the kth saddle point defined by the condition $(d/dz)g(z)|_{z=z_k} \equiv g^{(1)}(z_k)=0$. The branch of $\sqrt{-g^{(2)}(z_k)}$ ($g^{(2)}$ being the second derivative of g) is chosen in such a way that $\arg[\sqrt{-g^{(2)}(z_k)}]$ is equal to the angle between the positive direction of the tangent to the contour of integration in z_k and the positive direction of the real axis. We note that if $f(z)$ or $g(z)$ has singularities in the complex plane, further contributions may arise in Eq. (3) from the deformation of the integration path.

When applying the saddle point approximation to the integral in Eq. (1), special care has to be taken of the singularities in $S(k)$. To do so, we transform the integrand. In the first step, the dimensionless variables

$$\bar{x} = \frac{x}{a}, \quad \tau = \frac{t}{t_a}, \quad s = a(k-k_0), \quad l_i = ak_i, \quad \text{and } \rho_i = a\frac{\Gamma_i}{2}, \quad (4)$$

with $t_a = \frac{ma^2}{\hbar}$, are introduced. Here, we choose the width a of the unperturbed incident pulse at $t=0$ in real space as a reference length in the problem. This means that we "measure" the QS and other characteristics of the transmission in terms of the incident pulse size: In dimensionless quantities, the coordinate variable becomes $\bar{x}$, the time variable is τ, where t_a is the characteristic time of the pulse, the (shifted) momentum is s, the position of the poles of the S matrix and the center of the package position in momentum space are $l_{i\neq 0}$ and l_0, respectively, and the width of the ith resonance is ρ_i. Also, we introduce the following additional notation:

$$\gamma_i = \Delta_i + i\rho_i, \quad \Delta_i = l_0 - l_i, \quad \text{and } q = \bar{x} - l_0\tau. \quad (5)$$

With the new variables, we now write Eq. (1) as

$$\psi(\bar{x}>0,\tau)=\sum_{i}^{N}\psi_i(\bar{x},\tau)+\psi_b(\bar{x},\tau), \tag{6}$$

with

$$\psi_i(\bar{x},\tau)=i\frac{\rho_i S_0^i}{a\sqrt{2\pi}}\exp\left[il_0\left(\bar{x}-\frac{1}{2}l_0\tau\right)\right]I_i, \tag{7}$$

$$I_i=\int_{-\infty}^{\infty}ds\,\phi_0(s)\frac{1}{s+\gamma_i}\exp\left[iqs-i\frac{\tau}{2}s^2\right]=\int_{-\infty}^{\infty}ds\,\exp[g_i^{res}(s)], \tag{8}$$

$$g_i^{res}(s)=iqs-i\frac{\tau}{2}s^2-\log(s+\gamma_i)+\log[\phi_0(s)]. \tag{9}$$

Here, $\phi_0(s)=\psi_0(k-k_0)$ is centered around $s=0$. Furthermore,

$$\psi_b(\bar{x}>0,\tau)=i\frac{S_b}{a\sqrt{2\pi}}\exp\left[il_0\left(\bar{x}-\frac{1}{2}l_0\tau\right)\right]J, \tag{10}$$

with the integral $J=\int_{-\infty}^{\infty}ds\,\exp[g^{bg}(s)]$ and $g^{bg}(s)=iqs-i\frac{\tau}{2}s^2+\log[\phi_0(s)]$. Since the integral J can be gained from the limit $J=\lim_{\rho_i\to\infty}i\rho_i I_i$, we will focus on I_i in the following. As one can see, in variables (4) and (5), the size of the incident packet enters amplitude (7) as an overall factor. For actual calculations, it does not matter whether the packet is wide or narrow or whether the level of the QS is wide or narrow. All possible relations are incorporated on an equal footing.

We now apply the saddle point method [Eq. (3)] to obtain an approximation for I_i at finite $\lambda=1$. The motivation for this approach is that for each i, the integrand of the integrals in Eq. (8) might have a number of relevant saddle points s_{ik}^{res} denoted by the index k and determined by the condition $(d/ds)g_i^{res}|_{s_{ik}^{res}}\equiv g_i^{res(1)}(s_{ik}^{res})=0$. Apart from the "free phase" $iqs-i\tau s^2/2$, these maxima could also come from strong variations in $\phi_0(s)$ or $S(s+l_0)$. As a consequence, a number of saddle points,

$$s_{ik}^{res}=s_{ik}^{res}(l_0,l_i,\rho_i,\tau,q), \tag{11}$$

result. A multi-saddle-point approximation (MSA) is then obtained with

$$I_i\sim I_i^{MSA}=\sum_k \exp[g_i^{res}(s_{ik}^{res})]\sqrt{-\frac{2\pi}{g_i^{res(2)}(s_{ik}^{res})}}, \tag{12}$$

where $g_i^{res(2)}$ denotes the second derivative of $g_i^{res}(s)$. Combining the exponential form of the integrand in Eq. (6) and the dimensionless variable approach, one can apply the saddle point method with maximal efficiency. As pointed out in Eq. (11), the stationary points depend on a variety of parameters (l_0,l_i,ρ_i,τ,q) representing the intricate interplay between the incident pulse and the resonance. In contrast, if the integrals in Eq. (8) are calculated directly via the saddle point method setting in Eq. (3), $g(s)=iqs-i\frac{\tau}{2}s^2\equiv iqs-i\bar{\omega}(s)\tau$, a single saddle point,

$$s^0=\frac{q}{\tau}=s^0(q,\tau), \tag{13}$$

results, leading to a SSA. Note that s^0 contains no information about the resonance. On general grounds, one expects substantial corrections to the SSA if $S(s+l_0)$ varies strongly in the vicinity of s^0. The same happens if the incident pulse $\phi_0(s)$ is not a smooth function close to s^0. In the subsequent sections, we will consider as an explicit example an incident Gaussian reference pulse. We will demonstrate that, indeed, there are qualitative corrections to the SSA. These corrections consist of characteristic oscillations in the wave function of the transmitted pulse present in the analytical result but absent in the SSA. These oscillations can be described in the MSA where they result from the interference of the contributions stemming from different saddle points.

III. TRANSMISSION OF A GAUSSIAN PULSE

A. General results

We apply our approach to an incident Gaussian pulse,

$$\phi_0(s)=\psi_0\exp\left(-\frac{s^2}{2}\right). \tag{14}$$

Inserting Eq. (14) in Eq. (9) yields

$$g_i^{res}(s)=isq-\beta s^2+\log(\psi_0)-\log(s+\gamma_i), \tag{15}$$

with $\beta=(1+i\tau)/2$. The condition $g_i^{res(1)}=0$ leads to two saddle points, $k=1,2$, for each i which are given by

$$s_{i1/2}^{res}\equiv s_{1/2}=\frac{1}{2}\left(\frac{iq}{2\beta}-\gamma_i\right)\pm\frac{1}{2}\left[\left(\frac{iq}{2\beta}+\gamma_i\right)^2-\frac{2}{\beta}\right]^{1/2}. \tag{16}$$

We obtain the following from Eq. (12):

$$I_i^{MSA}=\exp[g_i^{res}(s_1)]\sqrt{-\frac{2\pi}{g_i^{res(2)}(s_1)}}+\exp[g_i^{res}(s_2)]\sqrt{-\frac{2\pi}{g_i^{res(2)}(s_2)}}, \tag{17}$$

with

$$g_i^{res(2)}(s)=-2\beta+\frac{1}{(s+\gamma_i)^2}, \tag{18}$$

from which ψ^{MSA} can be calculated according to Eq. (7) replacing $I_i\to I_i^{MSA}$.

Furthermore, for the particular case of the Gaussian pulse, one finds the analytic expression[13]

$$I_i=-i\pi\psi_0\exp(-\beta\gamma_i^2-iq\gamma_i)\mathrm{erfc}\left(\frac{q}{2\sqrt{\beta}}-i\gamma_i\sqrt{\beta}\right), \tag{19}$$

where $\mathrm{erfc}(x)$ is the complementary error function, defined as $\mathrm{erfc}(x)=1-\mathrm{erf}(x)$. As shown in Appendix B, the MSA and the analytical result agree exactly in the limit $\tau\to\infty$.

The SSA result is

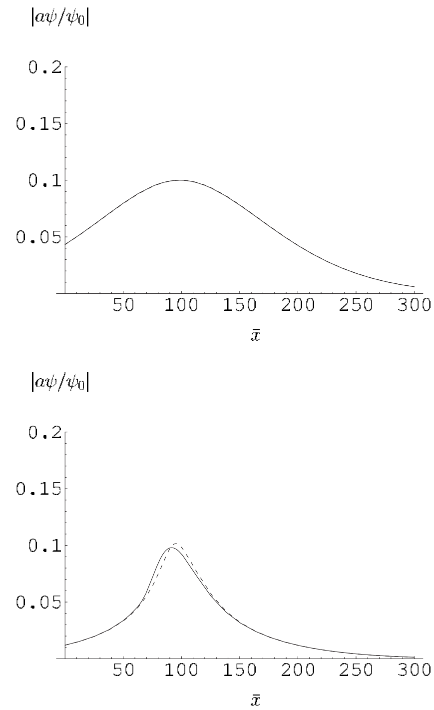

FIG. 2. $|\psi^{MSA}(\bar{x},\tau)|$ (dashed line) and $|\psi|$ (solid line) for $N=1$, $S_b=0$, $\tau=100$, $S_0^1=1.0$, $l_0=1.0$, $l_1=1.0$, $\rho_1=2.0$ (left hand side), and $\rho_1=0.2$ (right hand side).

$$I_i^{SSA} = \psi_0 \sqrt{\frac{2\pi}{i\tau}} \frac{1}{\frac{q}{\tau}+\gamma_i} \exp\left(-\frac{q^2}{2\tau^2}\right) \exp\left(i\frac{q^2}{2\tau}\right). \quad (20)$$

Equation (20) is derived using the saddle point approximation [Eq. (3)] for the integral in the first equality in Eq. (8), i.e., using the "free saddle point" in Eq. (13). With Eq. (7), the saddle point contribution is obtained by replacing $I_i \to I_i^{SSA}$,

$$|\psi_i^{SSA}| = \frac{|\psi_0|}{a} \frac{\rho_i |S_0^i|}{\sqrt{\tau}} \exp\left(-\frac{q^2}{2\tau^2}\right) \frac{1}{\sqrt{\left(\frac{q}{\tau}+\Delta_i\right)^2+\rho_i^2}}. \quad (21)$$

In leading order $1/\tau$, $\Delta=0$ and for $N=1$, the result for ψ_i^{SSA} is identical with the weakly distorted Gaussian pulse ψ_{WDG} in Eq. (10) of Ref. 13, $\psi_i^{SSA} \sim \psi_{WDG}$.

B. Numerical results

Representing the tunneling structure with Eq. (2) and the incident pulse with Eq. (14), the subsequent figures (Figs. 2–4) show the transmitted Gaussian pulse in MSA [Eqs. (7) and (17)] and the analytic result [Eq. (19)] for a number of cases with $N=1$. Reasonable agreement between $\psi_1^{MSA}=\psi^{MSA}$ and the analytical expression for ψ is obtained. The following general picture emerges: For sufficiently large values of ρ_1 ($\rho_1>0.2$, see Figs. 2 and 3), the contribution from s_2 is completely negligible and no oscillations are found. In this regime, the integrand in I_1 is a smooth function and the SSA and the MSA one well working approximations.

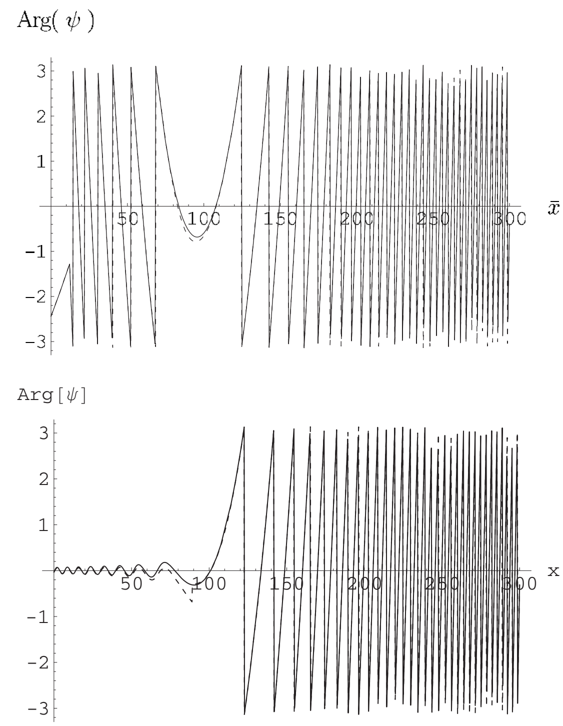

FIG. 3. $\arg(\psi^{MSA})$ (dashed line) and $\arg(\psi)$ (solid line) on the left hand side for $N=1$, $S_b=0$, $\tau=100$, $S_0^1=1.0$, $l_0=1.0$, $l_1=1.0$, $\rho_1=0.2$ (see right hand side of Fig. 3) and on the right hand side for $N=1$, $S_b=0$, $\tau=100$, $S_0^1=1.0$, $l_0=1.0$, $l_1=1.0$, $\rho_1=0.01$ (see left hand side of Fig. 5).

A weakly distorted Gaussian pulse $\psi^{SSA} \sim \psi_{WDG} \sim \psi$ results as already demonstrated in Ref. 13. In the opposite limit, for small ρ_1, the MSA is still working (see Fig. 4). The contribution from the saddle point s_1 still describes correctly the general behavior of the wave function, which has now changed to a strongly asymmetric decaying-state signal. Furthermore, the contribution of s_2 in the MSA represents an additional term whose interference with the contribution of s_1 leads to the characteristic oscillations seen in Fig. 4. In Ref. 13, it was derived that the decaying-state signal corresponds to a second component in the analytical wave function denoted with ψ_{DS} therein. Since ψ_{DS} is structurally different from $\psi_{WDG} \sim \psi^{SSA}$, the SSA fails to represent the decaying-state signal as well as the characteristic oscillations for a narrow resonance. Instead, the SSA gives a false weakly distorted Gaussian pulse.

Our numerical examples suggest that one can approximate the resonantly transmitted wave function resulting from the incident Gaussian pulse in the following way. For large values of ρ_i, the complete wave function can be approximated using the saddle point s_1 alone in Eq. (17). For small values of ρ_i, if we are interested only in the behavior of the signal amplitude averaged in space or time (which is most probably the case for a realistic detector), the wave function is again described well by expression (17) with only s_1 taken into account. However, if we are interested in the characteristic oscillations, then we have to take into account the small interfering contribution of the second critical point s_2 as well.

The presented method also allows us to adequately describe resonant quantum systems with a number of resonant

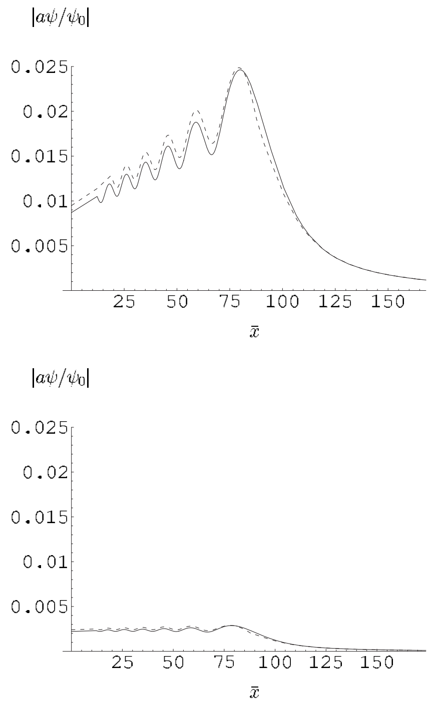

FIG. 4. $|\psi^{MSA}|$ (dashed line) and $|\psi|$ (solid line) for $N=1$, $S_b=0$, $\tau=100$, $S_0^1=1.0$, $l_0=1.0$, $l_1=1.0$, (a) $\rho_1=0.01$, and (b) $\rho_1=0.001$.

levels. Figures 5 and 6 show the absolute value of the transmitted pulse wave function in the case of a QS with two resonant levels for different values of ρ_i.

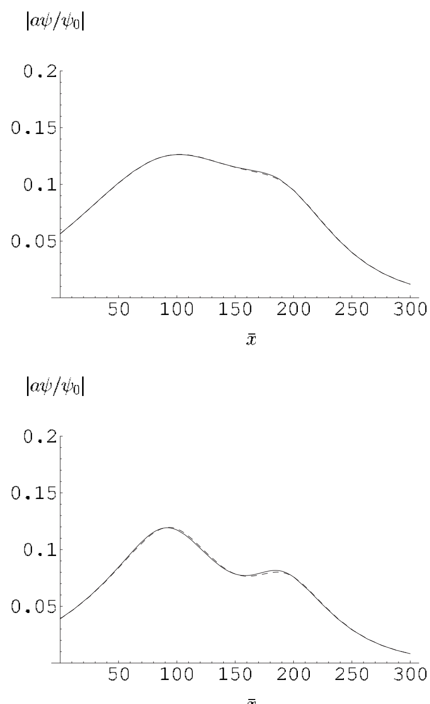

FIG. 5. $|\psi^{MSA}|$ (dashed line) and $|\psi_r^{ex}|$ (solid line) for $N=2$, $S_b=0$, $\tau=100$, $S_0^1=1.0$, $S_0^2=1.0$, $l_0=1.0$, $l_1=1.0$, $l_2=2.0$, (a) $\rho_1=1.0$, $\rho_2=0.5$, and (b) $\rho_1=0.5$, $\rho_2=0.4$.

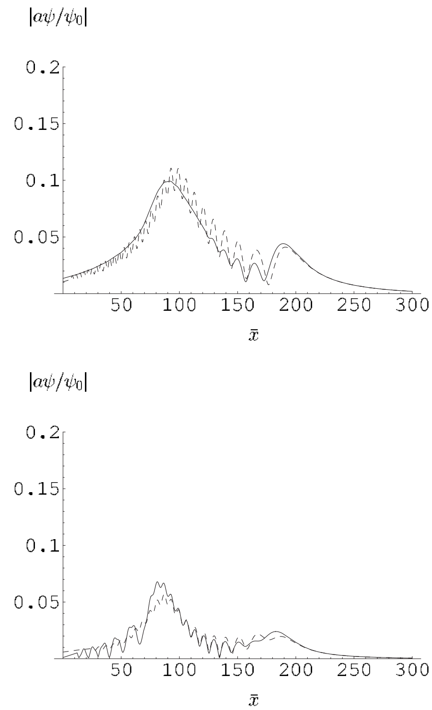

FIG. 6. $|\psi^{MSA}|$ (dashed line) and $|\psi|$ (solid line) for $N=2$, $S_b=0$, $\tau=100$, $S_0^1=1.0$, $S_0^2=1.0$, $l_0=1.0$, $l_1=1.0$, $l_2=2.0$, (a) $\rho_1=0.2$, $\rho_2=0.05$, and (b) $\rho_1=0.04$, $\rho_2=0.02$.

IV. CONCLUSIONS

We have developed a new asymptotic method for the solution of the problem on pulse propagation through a resonant QS described in the S-matrix approach. It is based on two ideas: First, a special choice of dimensionless variables describing either the system or the pulse and, second, the steepest descent method for the calculation of the involved integrals that admits to describe any relation between these variables. Hence, the information about the pulse and the QS enters uniquely into the saddle points and the path directions. This way, a good approximation for the transmitted pulse can be obtained. To verify the method, we have investigated the transmission of a Gaussian pulse through a resonant QS, establishing a good agreement with exact results derived already for this case.[13] We relate the minor difference between the exact and the approximate results (see Figs. 5 and 7) to the fact that in the latter case, only the first term of the asymptotic expansion was taken into consideration. To increase the accuracy, the next-to-leading terms of the expansion should be accounted for.

We also note that an interesting application of our method could be the calculation of the time delay. In fact, as shown in Appendix B, the exact expression for the transmitted amplitude and the approximate one practically coincide for a narrow resonance. This concerns the modulus as well as the phase factor (see Fig. 3). Since the delay time is expressed as the energy derivative of the transmitted phase, the derived analytic expression for the amplitude gives a possibility to calculate this parameter as an explicit function of all the relevant variables. This is a further advantage of our method.

The results on this problem, as well as a comparison with other approaches and investigation of other incident pulses, will be published elsewhere.

To complete, we say a few words about an issue of our consideration in general. It consists of the idea of relating intricate peculiarities and singular behavior of the amplitude with the saddle point positions. Then, due to analyticity, the regular part is described well by asymptotic formulas.

APPENDIX A: EXPANSION OF THE *S*-MATRIX IN A POLE REPRESENTATION

The S matrix is meromorphic in the entire complex k plane, leading to the general expansion[20,25,26]

$$S(k) = \sum_{i=-\infty}^{\infty} \left(\frac{R_i}{k - K_i} + \frac{R_i}{K_i} \right), \tag{A1}$$

with $R_i = iS_0^i \Gamma_i / 2$. In Eq. (2), we reduce Eq. (A1) to the special case of N isolated resonances. In this case, the contribution of all other resonances $i \ni \{1, \ldots, N\}$ can be approximated by a constant in the relevant k range so that

$$S_b \sim \sum_{i \ni \{1,2,\ldots,N\}} \frac{R_i}{k_0 - K_i} + \sum_{i=-\infty}^{\infty} \frac{R_i}{K_i}. \tag{A2}$$

Because of the summation over infinitely many K_i and R_i, it is difficult to construct S_b from Eq. (A2) directly. For $N=1$, one can calculate R_1, K_1, and S_b from the expression

$$S(E) \simeq i \frac{S(E_0) - S_{bg}}{e + i} + S_{bg}, \tag{A3}$$

with $e = 2(E - E_0)/\Gamma$, derived in Ref. 12. From Eq. (A3), $S(k)$ follows as given in Eqs. (4)–(6) of Ref. 13. However, in this paper, we consider the K_i, R_i, and S_b as given input parameters.

APPENDIX B: RESONANT TRANSMISSION OF A GAUSSIAN PULSE IN MUCH-SADDLE-POINT APPROXIMATION FOR $\tau \to \infty$

Let us write explicitly the expression for the saddle point s_1 in Eq. (16) in the limit of large τ. We find the following for the leading order terms:

$$s_1 = i \frac{\bar{x} - l_0 \tau}{1 + i\tau} + \frac{1}{l_i - l_0 + \rho_i \tau - i(\bar{x} + \rho_i - l_i \tau)} + O(\beta^{-2}). \tag{B1}$$

Taking into account only the leading order term in $\tau \to \infty$, it results that

$$|g^{res(2)}(s_1)|^{-1/2} = |1 + \tau^2|^{1/4} |s_1|^{1/4} = |1 + \tau^2|^{1/4} + O(\tau^{-2}) \tag{B2}$$

and

$$\frac{1}{|s_1 + \gamma_i|} = \frac{\sqrt{1 + \tau^2}}{[(l_i - l_0 + \rho_i \tau)^2 + (\bar{x} + \rho_i - l_i \tau)^2]^{1/2}}. \tag{B3}$$

Substituting these expressions into Eqs. (17) and (8), setting $S_b = 0$, and taking into account only a single i, we find

$$|\psi^{MSA}(\bar{x}, \tau \to \infty)| = C' e^{-(1/2)[(\bar{x} - l_0 \tau)^2 / 1 + \tau^2]} \times \frac{(1 + \tau^2)^{1/4}}{[(l_i - l_0 + \rho_i \tau)^2 + (\bar{x} + \rho_i - l_i \tau)^2]^{1/2}} \tag{B4}$$

for the modulus of the transmitted pulse where C' is an inessential constant. This expression agrees with the corresponding leading order term of the analytic result in Eq. (19) which can be written in the form

$$\psi(\bar{x}, \tau \to \infty) = C \exp[i \arg(\psi_{as})] e^{-(1/2)[(\bar{x} - l_0 \tau)^2 / 1 + \tau^2]} \times \frac{(1 + \tau^2)^{1/4}}{[(\bar{x} + \rho_i - l_i \tau)^2 + (\rho_i \tau + l_i - l_0)^2]^{1/2}}, \tag{B5}$$

with

$$\arg(\psi_{as}) = \frac{1}{2} \arctan \tau + \frac{(\bar{x} - l_0 \tau)^2 \tau}{2(1 + \tau^2)} - \arctan\left[\frac{\rho_i \tau + l_i - l_0}{\bar{x} + \rho_i - l_i \tau} \right]. \tag{B6}$$

The contribution to the wave function coming from the saddle point s_2 is damped in time much faster than that of s_1 and can therefore be neglected at long times.

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